

A Short Talk About A Long Way:  
IR Behavior Of Yang Mills Theory In Maximally  
Abelian Gauge  
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## Outline.

- 1 Fixing The Gauge: Landau Gauge
- 2 The Maximally Abelian Gauge
- 3 An Outlook

# The Journey Begins.

## A Starting Point.

The QCD Lagrangian.

$$\mathcal{L} = \frac{1}{4} F_{\mu\nu} F_{\mu\nu} + \mathcal{L}_{matter}, \quad (1)$$

with

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i[A_\mu, A_\nu]. \quad (2)$$

- 1 Yang-Mills: gluonic part
- 2 Action invariant under rotations in color space
- 3 Equivalent field configurations
- 4 Count one representative
- 5 Fix the gauge

## Steering The Course: Where To Go?

Fixing the gauge in YM-Theory

- 1 Path integral overcomplete
- 2 Gauge orbits (set of field configurations connected by a gauge transformation):

$$O[A] := \{A'_\mu \mid A'_\mu = A_\mu^U\}, \quad (3)$$

with

$$A_\mu^U(x) = U(x)A_\mu(x)U(x)^{-1} + \frac{i}{g}(\partial_\mu U(x))U(x)^{-1}, \quad (4)$$

and  $U(x) = e^{ig\omega(x)}$  and  $\omega(x)$  gauge parameter

- 3 Faddeev-Popov: restrict integration of path integral
- 4 Ghosts: even spin, but fermionic

# Heading For New Horizons (e.g. Gribov's).

## Setting Sail...

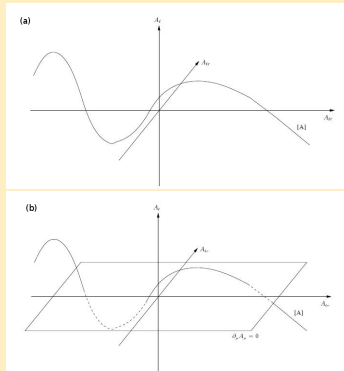


Figure: Gauge Orbits and Gribov Copies, (M. Huber)

## ...For Deep Waters.

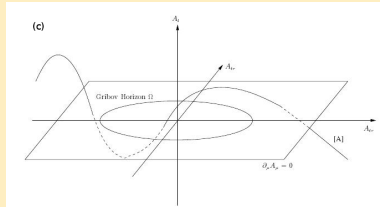


Figure: One gauge-configuration per orbit: The Gribov region, (M. Huber)

- (a): A gauge orbit in field configuration space
- (b): Restrict integration to a hyperplane,  $\partial A = 0$
- (c): Gauge orbit should intersect hyperplane  $\partial A = 0$  only once

# Points Of Interest Along The Way.

## Some Remarks On The Gribov-Region.

- 1 Gribov region is bounded (not true in MAG)
- 2 Every gauge orbit passes through the Gribov region at least once
- 3 Still gauge copies within the region, expectation value not influenced by them
- 4 Fundamental Modular Region

## Gauge Fixing.

Functional integration  $\int [dA]$  should count one representative of a set of gauge copies. Therefore, restriction to hyperplane  $\partial_\mu A_\mu = 0$ .

## That Place Is Haunted.

- Introduce Grassmannfields  $c, \bar{c}$  (*ghosts*) for restriction
- Ghost fields interact with gluons

# Yang-Mills in Landau Gauge.

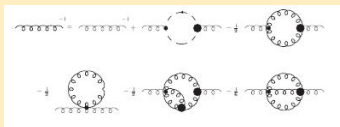
## Building Blocks Of The New Lagrangian.

The Lagrangian consists now of:

- $AA$ ,  $cc$ , propagators
- $AAA$ ,  $Acc$ , 3-point interaction
- $AAAA$ , 4-point interaction

## The DSE Of Yang Mills In Landau Gauge.

Dyson Schwinger Equations  
of Landau Gauge Yang Mills  
Theory, (M. Huber,  
arXiv:1005.1775v1 [hep-th]  
(2010))



# Reaching The Destination - The Playground.

## The Maximally Abelian Gauge.

Identifying the Abelian part of the gauge field.

Generators of the algebra:

$$[T^r, T^s] = if^{rst} T^t, \quad (5)$$

with  $f^{rst}$  the structure constants. Abelian part of an algebra: The Cartan subalgebra is the maximal set of commuting generators:

$$[T^i, T^j] = 0. \quad (6)$$

Matrices for generators chosen diagonal, therefore *diagonal* and *off-diagonal* part.

## The Gauge Field.

The gauge field can now be separated into diagonal and off-diagonal part:

$$A_\mu = T^i A_\mu^i + T^a B_\mu^a. \quad (7)$$

## Building Blocks in the Lagrangian.

We now have the following diagonal and off-diagonal interactions in the Lagrangian:

- BBBB
- ABBB
- AABB
- ABB
- BBB

# Why Going On This Odysee?

## Gauge Fixing.

Due to gauge fixing ghosts are introduced, giving rise to DSE building blocks:

- ABB
- Acc
- AABB
- AAcc
- BBcc
- BBBB
- CCCC

## Why using MAG?

- 1 IR regime of MAG
- 2 Confinement
- 3 Dual superconductor
- 4 Hypothesis of Abelian IR dominance (Z. Ezawa, A. Iwazaki, Phys. Rev. D25 2681 (1982))

## Behavior in Landau and MA gauge.

- 1 Green functions are gauge-dependent
- 2 IR Leading Terms
  - Landau gauge: ghost fields
  - MA gauge: abelian gluons

This is in agreement with the hypothesis of Abelian dominance.



# DSEs Of Yang Mills In MAG.

## Gluons.

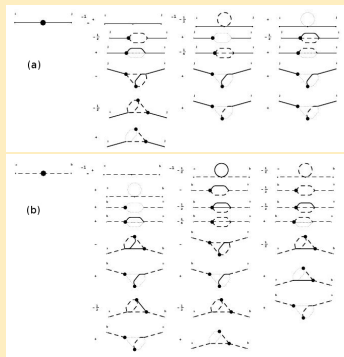


Figure: DoDSE output for diagonal (a) and off-diagonal (b) gluon 2-point functions, (M. Huber, arXiv:1005.1775v1 [hep-th] (2010))

## Ghosts.

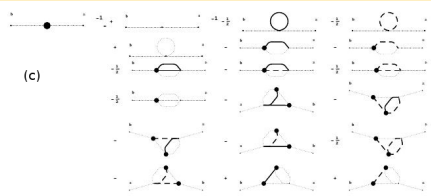
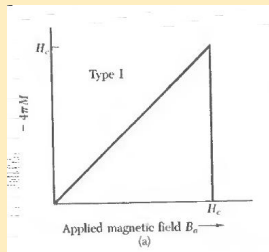


Figure: DSE of ghost two-point function (M. Huber, arXiv:1005.1775v1 [hep-th] (2010))

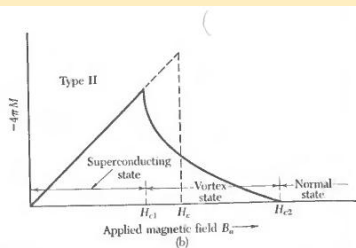
- continuous line: diagonal gluon
- dashed line: off-diagonal gluon
- dotted line: ghost

# Superconducting Magnetization Curves.

## Type I Superconductor.



## Type II Superconductor.



C. Kittel, Introduction To Solid State Theory

# The Dual Superconductor Picture.

## Abrikosov Vortices And The Meissner Effect.

Magnetic fields in type II superconductors are collimated into magnetic flux tubes, the Abrikosov vortices. Monopole-antimonopole pair connected by a fluxtube, growth of energy stored in field would be linear with separation.  
(R. Alkofer, J. Greensite, J. Phys. G: Nucl. Part. Phys. 34, S3-S21 (2007))

### Type II Superconductor

magnetic charge  
confined by a  
magnetic flux tube  
in a condensate of  
electrically charged  
objects

### ← Duality →

↔

↔

↔

### Dual Superconductor

color electrical charge  
confined by an  
electric flux tube  
in a condensate of  
magnetically charged  
objects

## Outlook.

The working title of my diploma thesis is:

*"Infrared consistent truncation for Green functions of Yang Mills Theory in maximally Abelian gauge"*

# To Be Continued...

Thank you...

... for your attention!